$\boxed{1}$ أوجدي $\frac{dy}{dx}$ لكل من الدوال الآتية:

$\boxed{1}$ $y = \cosh^2(3x)$

$$\frac{dy}{dx} = 2 \cdot \cosh(3x) \cdot \sinh(3x) \cdot 3$$

$$= 6\cosh(3x) \cdot \sinh(3x) \quad \#$$

$\boxed{2}$ $\sinh y = \sec x$

$$\therefore y = \sinh^{-1}(\sec x)$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{\sec^2 x + 1}} \cdot (\sec x \cdot \tan x) \quad \#$$

$\boxed{3}$ $y = \tanh(\ln x)$

$$\frac{dy}{dx} = \text{sech}(\ln x) \cdot \frac{1}{x} \quad \#$$

$\boxed{4}$ $y = \tanh^{-1}\left(\frac{x}{a}\right)$

$$\frac{dy}{dx} = \frac{1}{1-\left(\frac{x}{a}\right)^2} \cdot \frac{1}{a}$$

$$= \frac{a}{a^2 - x^2} \quad \#$$

[5] أوجد تكامل كل من الدوال الآتية:-

[1] $\int_0^2 \frac{dx}{9-x^2}$

$= \frac{1}{9}\int_0^2 \frac{dx}{1-\frac{x^2}{9}} = \frac{1}{3}\tanh^{-1}\left(\frac{x}{3}\right)\Big|_0^2$

= by calculator

[2] $\int \frac{\sinh\sqrt{x}}{\sqrt{x}}\,dx$

$= 2\int \frac{\sinh\sqrt{x}}{2\sqrt{x}}\,dx = 2\cosh(\sqrt{x}) + C$

[3] $\int e^x \sinh(2x)\,dx$

where $\sinh x = \frac{e^x - e^{-x}}{2}$

$\therefore \sinh 2x = \frac{e^{2x} - e^{-2x}}{2}$

$\therefore I = \int e^x\cdot\left(\frac{e^{2x}-e^{-2x}}{2}\right)dx$

$= \frac{1}{2}\int \left(e^{3x} - e^{-x}\right)dx$

$= \frac{1}{2}\left[\frac{1}{3}e^{3x} + e^{-x}\right] + C$ #

أثبتي الصيغة الآتية :

$$\operatorname{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$$

Proof:

Let $y = \operatorname{sech}^{-1} x$

$\therefore \quad x = \operatorname{sech} y = \dfrac{1}{\cosh y}$

where $\cosh y = \dfrac{e^{y}+e^{-y}}{2}$

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$\therefore \quad x = \dfrac{2}{e^{y}+e^{-y}}$

$\therefore \quad x e^{y} + x e^{-y} = 2 \qquad (* e^{y})$

$\therefore \quad x e^{2y} + x - 2e^{y} = 0$

$\therefore \quad e^{y} = \dfrac{+2 \pm \sqrt{4-4x^2}}{2x} = \dfrac{1 \pm \sqrt{1-x^2}}{x}$

$\therefore \quad y = \ln\left(\dfrac{1+\sqrt{1-x^2}}{x}\right)$

$\therefore \quad \operatorname{sech}^{-1} x = \ln\left(\dfrac{1+\sqrt{1-x^2}}{x}\right)$ #

$\boxed{٤}$ إذا كانت $\cosh x = \frac{17}{15}$ أوجد بقية الدوال :

where $\cosh^2 x - \sinh^2 x = 1$

$\therefore \sinh^2 x = \cosh^2 x - 1 = \frac{289}{225} - 1$

$= \frac{64}{225}$

$\therefore \sinh x = \frac{8}{15}$

$\therefore \tanh x = \frac{\sinh x}{\cosh x} = \frac{8}{17}$

$\therefore \coth x = \frac{1}{\tanh x} = \frac{17}{8}$

$\therefore \operatorname{sech} x = \frac{1}{\cosh x} = \frac{15}{17}$

$\therefore \operatorname{cosech} x = \frac{1}{\sinh x} = \frac{15}{8}$

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#

[٥] أوجد التكاملات الآتية:

[1] $\int \frac{\sin x}{1-\cos^2 x}\, dx$

$= \tanh^{-1}(\cos x) + C$

[2] $\int \operatorname{sech}^2(5x)\, dx$

$= \frac{1}{5}\int 5 \operatorname{sech}^2(5x)\, dx = \frac{1}{5}\tanh(5x) + C$

[3] $\int \sinh x \operatorname{sech}^2 x\, dx$

$= \int \sinh x \cdot \frac{1}{\cosh^2 x}\, dx = \int \cosh^{-2} x \cdot \sinh x\, dx$

$= -\frac{1}{\cosh x} + C = -\operatorname{sech} x + C$ #

[7] باستخدام التعريف الأساسي للدالة $(\sinh x)$ اثبت أن :

$$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y.$$

Proof:

$$\sinh x = \frac{e^{x}-e^{-x}}{2} \quad \& \quad \sinh y = \frac{e^{y}-e^{-y}}{2}$$

$$\cosh x = \frac{e^{x}+e^{-x}}{2} \quad \& \quad \cosh y = \frac{e^{y}+e^{-y}}{2}$$

Then

$$R.H.S. = \sinh x \cosh y + \cosh x \sinh y$$

$$= \frac{e^{x}-e^{-x}}{2} \cdot \frac{e^{y}+e^{-y}}{2} + \frac{e^{x}+e^{-x}}{2} \cdot \frac{e^{y}-e^{-y}}{2}$$

$$= \frac{1}{4}\left[e^{x+y} \cancel{- e^{y-x}} \cancel{+ e^{x-y}} - e^{-(x+y)} \right]$$

$$+ \frac{1}{4}\left[e^{x+y} \cancel{+ e^{y-x}} \cancel{- e^{x-y}} - e^{-(x+y)} \right]$$

$$= \frac{e^{x+y} - e^{-(x+y)}}{2}$$

$$= \sinh(x+y) = L.H.S. \quad \#$$

$\boxed{V}$ أوجد التكاملات المعتلة

$\boxed{1}$ $\displaystyle\int_0^2 \frac{1}{x^2} \cos\left(\frac{1}{x}\right) dx$

Soln:-

$$I = \lim_{t \to 0^+} \int_t^2 \frac{1}{x^2} \cos\left(\frac{1}{x}\right) dx$$

$$= -\lim_{t \to 0^+} \int_t^2 \frac{-1}{x^2} \cos\left(\frac{1}{x}\right) dx$$

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$$= -\lim_{t \to 0^+} \left[\sin\left(\frac{1}{x}\right)\right]_t^2$$

$$= -\lim_{t \to 0^+} \left[\sin\left(\frac{1}{2}\right) - \sin\left(\frac{1}{t}\right)\right]$$ does not Converge.

$$\int^{\frac{\pi}{2}} \sec^2 x \, dx$$

:

$$I = \lim_{t \to \frac{\pi}{2}^-} \int_{\frac{\pi}{4}}^{t} \sec^2 x \, dx$$

$$= \lim_{t \to \frac{\pi}{2}} \left[\tan x\right]_{\frac{\pi}{4}}^{t}$$ = does not Converge

$\boxed{3}$ $\displaystyle\int_0^4 \frac{dx}{x^2+x-6}$

$x^2+x-6=0$

$(x-2)(x+3)=0$

$x=2$ or $x=-3$

$x=2 \in [0,4]$

Soln:

$$I = \int_0^2 \frac{dx}{x^2+x-6} + \int_2^4 \frac{dx}{x^2+x-6}$$

$$= \int_0^2 \frac{dx}{(x-2)(x+3)} + \int_2^4 \frac{dx}{(x-2)(x+3)}$$

Where

$$\frac{1}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3}$$

$$1 = A(x+3) + B(x-2)$$

$x=2 \Rightarrow 1 = 5A \Rightarrow A = \frac{1}{5}$

$x=-3 \Rightarrow 1 = -5B \Rightarrow B = -\frac{1}{5}$

$$\therefore I = \frac{1}{5} \lim_{t \to 2^-} \int_0^t \left[\frac{1}{x-2} - \frac{1}{x+3}\right] dx$$

$$+ \frac{1}{5} \lim_{\alpha \to 2^+} \int_\alpha^4 \left[\frac{1}{x-2} - \frac{1}{x+3}\right] dx$$

$$= \frac{1}{5} \lim_{t \to 2^-} \left(\ln \frac{x-2}{x+3}\right)_0^t + \frac{1}{5} \lim_{\alpha \to 2^+} \left(\ln \frac{x-2}{x+3}\right)_\alpha^4$$

[٨] ادرس تقارب وتباعد التكاملات المعتلة الآتية:

[1] $\int_0^{\infty} \frac{dx}{1+x^2}$

Soln:

$$I = \lim_{t \to \infty} \int_0^{t} \frac{dx}{1+x^2}$$

$$= \lim_{t \to \infty} \left[\tan^{-1}(x) \right]_0^t = \lim_{t \to \infty} \left[\tan^{-1} t - \tan^{-1}(0) \right]$$

$$= \tan^{-1}\left(\frac{1}{0}\right) - \tan^{-1}\left(\frac{0}{1}\right)$$

$$= \frac{\pi}{2} - 0 = \frac{\pi}{2} \quad \# \text{ Converge}$$

[2] $\int_0^{1} \frac{dx}{\sqrt{1-x^2}}$

Proof:

$$I = \lim_{t \to 1^-} \int_0^{t} \frac{dx}{\sqrt{1-x^2}}$$

$$= \lim_{t \to 1^-} \left[\sin^{-1}(x) \right]_0^t = \lim_{t \to 1^-} \left[\sin^{-1}(t) - \sin^{-1}(0) \right]$$

$$= \sin^{-1}(1) = \frac{\pi}{2} \quad \# \text{ Converge}$$

$\boxed{3}$ $\displaystyle\int_0^{\pi/2} \tan x \, dx$

__Soln:__

$$I = \lim_{t \to \frac{\pi}{2}^-} \int_0^t \frac{\sin x}{\cos x} \, dx$$

$$= -\lim_{t \to \frac{\pi}{2}^-} \int_0^t \frac{-\sin x}{\cos x} \, dx$$

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$$= -\lim_{t \to \frac{\pi}{2}^-} \Big[\ln |\cos x| \Big]_0^t$$

$$= -\lim_{t \to \frac{\pi}{2}^-} \Big[\ln |\cos t| - \ln(1) \Big]$$

$$= \infty \qquad \# \ \underline{\text{divergence}}$$

$\boxed{9}$ اثبت أن التكامل $\displaystyle\int_{-1}^{7} \frac{dx}{(x+1)^{?}}$ يتقارب وأوجد قيمته.

التكامل يرجع [illegible] ١٢ [illegible] كرهم

الأرقام ([illegible]) [illegible]

1 اختر الإجابة الصحيحة مع التوضيح :

1 $\int_1^{\infty} x^{-2}\, dx$

Soln:

$$I = \lim_{t \to \infty} \int_1^t \frac{1}{x^2}\, dx$$

$$= -\lim_{t \to \infty} \left(\frac{1}{x}\right)_1^t = -\lim_{t \to \infty} \left(\frac{1}{t} - 1\right)$$

$$= +1 \quad \# \ \text{Ans: (i)}$$

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2 $\int_1^{\infty} \sqrt{x}\, dx$

Soln:

$$I = \lim_{t \to \infty} \int_1^t x^{1/2}\, dx$$

$$= \frac{2}{3} \lim_{t \to \infty} \left[x^{3/2}\right]_1^t = \frac{2}{3} \lim_{t \to \infty} \left(t^{3/2} - 1\right)$$

$$= \infty \quad \# \ \text{Ans: (iv)}$$

$\boxed{3}$ $\int_0^\infty x^{1/2}\,dx$

Soln:

$$I = \lim_{t\to\infty} \int_0^t x^{1/2}\,dx$$

$$= \frac{2}{3}\lim_{t\to\infty}\Big[x^{3/2}\Big]_0^t = \frac{2}{3}\lim_{t\to\infty}\left(t^{3/2} - 0\right)$$

$$= \infty \quad \# \quad \text{Ans: (iv)}$$

$\boxed{4}$ $\int_1^\infty \frac{\ln x}{x}\,dx$

Soln:

$$I = \lim_{t\to\infty} \int_1^t \frac{\ln x}{x}\,dx$$

$$= \frac{1}{2}\lim_{t\to\infty}\Big[(\ln x)^2\Big]_1^t$$

$$= \frac{1}{2}\lim_{t\to\infty}\Big[(\ln t)^2 - 0\Big]$$

$$= \infty \quad \# \quad \text{Ans: (iv)}$$

$\boxed{5}$ $\displaystyle\int_0^2 \frac{dx}{2x-3}$

$2x-3=0$
$2x=3 \Rightarrow x=\frac{3}{2}$
$x=\frac{3}{2}\in[0,2]$

<u>Soln:-</u>

$$I=\int_0^{3/2}\frac{dx}{2x-3}+\int_{\frac{3}{2}}^{2}\frac{dx}{2x-3}$$

$$=\lim_{t\to\frac{3}{2}^-}\int_0^t\frac{dx}{2x-3}+\lim_{\alpha\to\frac{3}{2}^+}\int_\alpha^2\frac{dx}{2x-3}$$

$$=\frac{1}{2}\lim_{t\to\frac{3}{2}^-}\Big[\ln|2x-3|\Big]_0^t+\frac{1}{2}\lim_{\alpha\to\frac{3}{2}^+}\Big[\ln|2x-3|\Big]_\alpha^2$$

$$=\frac{1}{2}\lim_{t\to\frac{3}{2}^-}(\ln 2t-3) \rightsquigarrow \text{undefined}$$

Ans: (iv)